INTUITIONISTIC CONNECTED FUZZY DOMINATING, INDEPENDENT & TOTAL STRONG (WEAK) DOMINATING SET

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ABSTRACT:

The basic definitions of intuitionistic fuzzy graph, intuitionistic connected independent & dominating fuzzy graph, intuitionistic total strong (weak) connected dominating sets and intuitionistic connected fuzzy graph are discussed.

The aim of this paper is to find on what conditions the intuitionistic connected fuzzy graph has intuitionistic equal domination number and total strong (weak) connected dominating number and independent connected dominating number. It is discussed briefly and also when the intuitionistic connected domination perfect is proved. Finally, the intuitionistic independent domination number for a connected intuitionistic fuzzy graph is obtained.

KEYWORDS:

Intuitionistic connected dominating set, intuitionistic independent set, intuitionistic total strong (weak) dominating set, intuitionistic connected independent& Total strong (weak) domination number.

1. INTRODUCTION:

Given a graph, a dominating set is a subset D of nodes such that each node in the graph is either in D, or has a direct neighbor in D. The theory of domination has been the nucleus of research activity in graph theory in recent times. The use of domination in graph lies in various fields in solving real life problems, it includes social network theory, Radio station, School Bus Routing, Land Surveying, computer communication networks, and also in the field of medicine. The problem of utilizing the online social network for solving social problem in the physical world such as drinking, smoking and drug problem are all explored well. The dominating set plays a vital role in investigating the mobile device in wireless mode; it has been commonly used for routing and broadcastings information to the mobile devices in mobile ad-hoc networks. Using graph theory as a modelling tool in biological networks allows the utilization of the most graphical invariants in such a way that it is possible to identify secondary RNA (Ribonucleic acid) motifs numerically. Those graphical invariants are variations of the domination number of a graph. Hence, the domination problem, along with many its variations play a significant role in graph theory.

Dominating sets are also used as models in operational research such as facility location problems. Multiple domination can be used to construct hierarchical overlay networks in peer-to-peer applications for more efficient index searching. The hierarchical overlay networks usually serve as distributed databases for index searching, substantial and quickly developing applications of multiple domination in modern computer networks is a wireless sensor network.

The intuitionistic fuzzy set is introduced by Atanassov as a generalization of fuzzy set,which have both membership grades and non-membership grades. Moreover, he has also introduced and studied many types of operations, relations, and its properties, which were analogs to fuzzy sets. As an application, he applied his idea into expert systems, pattern recognition and especially in decision-making. As a new emerging study of an intuitionistic fuzzy graph (IFG) has been addressed. Chountas and Alzebdi presented an intuitionistic fuzzy version of a tree in graph theory. Furthermore, the operations and some particular case of intuitionistic fuzzy graphs were done by Parvathy and Karunambigai. The domination of graph in an intuitionistic environment defined in, Later, Nagoorgani and Devi proposed the idea of edge domination and independence in the fuzzy graph. In this research, inspired, if investigated the concept of edge dominating set in IFGs and proved some remarkable results on edge dominating set. If produce the some theorems of total strong (weak) dominating intuitionistic Fuzzy graph.

2. PRELIMINARIES:

In this section, some basic definitions relating to ICFGs are given.

Also the definition of connected dominating set, cardinality of independent dominating set, total strong(weak)&degree of vertex and minimal connected domination in IFGs are studied.

Definition 2.1:

An **Intuitionistic Fuzzy set** [IFS] A in a universal set X is

defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$,

Where $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ denote the degree of membership and the

degree of non-membership of the element x in X respectively,

satisfying $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.2:

An **Intuitionistic Fuzzy sets of second type** [IFSST] A in a universal set

X is defined as an object of the form $= \{\langle x, {}_A(x), \nu_A(x)\rangle / x \in X\}$, Where μ_A:X→[0,1] and ν_A:X→[0,1] denote the degree of membership and the

degree of non-membership of the element x in X respectively,

satisfying $0\leq\mu_A(x)^2+\nu_A(x)^2\leq1$.

Definition 2.3:

An **Intuitionistic Fuzzy Graph** [IFG] is of the form $G=[V,E]$ where

(i) $V=\{v_1,v_2,\ldots v_n\}$ such that $\mu_1:V\to[0,1]$ and $\nu_1:V\to[0,1]$ denote the degree of membership and non-membership of the element $v_i\in V$, respectively, and $0\leq\mu_1(v_i)+\nu_1(v_i)\leq1$ for every $v_i\in V,(i=1,2,\ldots n)$

(ii) $E\subseteq V\times V$ where $\mu_2:V\times V\to[0,1]$ and $\nu_2:V\times V\to[0,1]$ are such that

$\mu_2(vi,vj)\leq\min[\mu_1(v_i),\mu_1(v_j)]$,

$\nu_2(v_i,v_j)\leq\max[\nu_1(v_i),\nu_1(v_j)]$

and $0\leq\mu_2(v_i,v_j)+\nu_2(v_i,v_j)\leq1$ for every $(v_i,v_j)\in E,(i,j=1,2,\ldots n)$

FOR EXAMPLE:

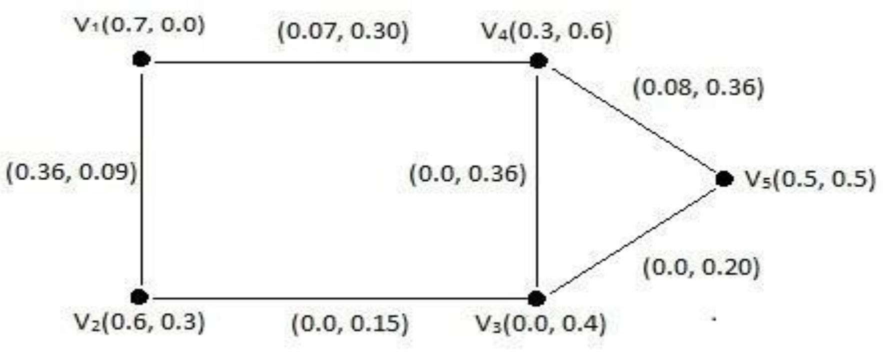

Fig.1:INTUITONISTIC FUZZY GRAPH

Definition 2.4:

The **strong neighbourhood** of an edge e_i in a intuitionistic fuzzy graph G is $(e_i)=\{e_j\in E(G)\,/e_j$ *is a strong arc in G and adjacent to* $e_i\}$ $(S\subseteq E(G))$ and $0\leq\mu_2(v_i,v_j)+\nu_2(v_i\ ,v_j)\leq1$ for every $(v_i,v_j)\in E,(i,j=1,2,\ldots n)$

Definition 2.5:

Let $G=(V,)$ be an intuitionistic fuzzy graph. Let e_i *and* e_j be two edges of G. If say that $_i$ dominates e_j, if e_i is a **strong arc** in G and adjacent to e_j.

Definition 2.6:

Let D be a **minimum dominating set of intuitionistic fuzzy graph** G. If for every $e_j \in (G) - D$, there exists $e_i \in D$ such that e_i dominates e_j, then D is called an edge dominating set of D . The minimum intuitionistic fuzzy cardinality of all edge dominating set of intuitionistic fuzzy graph G is known as edge domination number and it is denoted by (G).

Definition 2.7:

Let $G = (V, E)$ *be an IFG on V. Let* $u, v \in V$*, we say that u* ***dominates*** *v in G if there exists a strong edge between them.*

Definition 2.8.

A ***subset*** *S of V is called dominating set in G if for every* $v \in V - S$ *, there exists* $u \in S$ *such that u dominates v .*

Definition 2.9:

A dominating set S of an IFG is said to be **connected minimal dominating set** if no proper subset of S is a dominating set.

Definition 2.10:

Let u be a vertex in an IFG, then the **neighbourhood** of u is represent by $N(u) = \{v \in V / (u, v)$ *is a strong arc*$\}$.

Definition 2.11:

A vertex $u \in V$ of an IFG $G = (V, E)$ is said to be an **isolated vertex** if $\mu_2(u,v)=0$ and $\nu_2(u,v)=0$ for all $v \in V$. That is , $N(u) = \varphi$. From the definition, we came to know that an isolated vertex was never dominating any other vertex in G.

Definition 2.12:

An independent set S of G in an IFG is said to be **maximal independent** , if for every vertex $v \in V - S$, the set $S \cup \{v\}$ is not independent.

Definition 2.13:

The minimum cardinality among all maximal independent set is called **lower independence number** of G, and it is denoted by $i(G)$.

Definition 2.14:

The maximum cardinality among all maximal independent set is called **upper independence number** of G, and it is denoted by $I(G)$.

Definition 2.15:

A vertex $v \in G$ is said to be end-vertex of IFG, if it has at most one **Strong neighbor** in *G*.

Definition 2.16:

Let *G = (V, E)* be an IFG. Then the cardinality of *G* is defined to be

$$|G| = |\sum_{vi\in V}\frac{1+\mu 1(vi)-\vartheta 1(vi)}{2} + \sum_{vi,vj\epsilon V}\frac{1+\mu 2(vi,vj)-\vartheta 2(vi,vj)}{2}|$$

Definition 2.17:

A dominating set D of IFG is said to be minimal dominating set if no proper subset of S is a dominating set. Minimum Cardinality among all minimal dominating set is called the **Intuitionistic fuzzy domination Number**, and is denoted by γ_{if} (G).

3. SOME THEOREMS OF ICFD AND INDEPENDENT, STRONG (WEAK) DOMINATING SET:

Definition 3.1:

Two nodes that are joined by a path are said to be **connected**. The relation connected IFG is reflexive, symmetric, and transitive this relation satisfied.

Definition 3.2:

Let $G=(V,E)$ be an intuitionistic fuzzy graph. Let e_i *and* e_j be two edges of G. If say that e_i **dominates** e_j, if e_i is a strong arc in G and adjacent to e_j.

Definition 3.3:

Let D be a **minimum dominating set of intuitionistic fuzzy graph** G. If for every $e_j\in(G)-D$, there exists $e_i\in D$ such that e_i dominates e_j, then D is called an edge dominating set of D . The minimum intuitionistic fuzzy cardinality of all edge dominating set of intuitionistic fuzzy graph G is known as edge domination number and it is denoted by $\gamma_e(G)$.

Definition 3.4:

Let $G=(\sigma,\mu)$ be a fuzzy graph. Let e_i and e_j be two edges of G. if say that e_i dominates e_j if e_i is a strong arc in G and adjacent to e_j. Let $D\subseteq(G)$ and $\forall$ $e_j\in E(G)-D$, $\exists$ $e_i\in D$ such that e_i dominates e_j. Now we can say that D is an edge dominating set of G. Two edges e_i and e_j are said to be fuzzy independent, if $e_i\notin(e_j)$ *and* $e_j\notin Ns(e_i)$. A set $S\subseteq(G)$ is said to be a fuzzy edge independent set of G, if any two edges in S are **fuzzy independent**.

Definition 3.4:

A point dominating set is $D\subseteq V(G)$ of any IFG 'G' is a connected point set of dominating of G if the subgraph < D > induced by D is a connected intuitionistic fuzzy graph. The minimum cardinality taken over all minimal connected point set of dominating set is called the **connected point set of IFD** number $\gamma_{cp}(G)$.

Definition 3.5:

Let ICFG G=(V,E) and D is an Vertex dominating set in G.

- If the induced subgraph ⟨V−D ⟩ is disconnected. Then D is called split vertex dominating set of G.
- If the induced subgraph ⟨V−D⟩is connected. Then D is called a non-split vertex dominating set of G.
- If the induced subgraph ⟨V−D⟩ is a path. Then D is called a path non-split vertex dominating set of G.
- If the induced subgraph ⟨V−D⟩is a cycle. Then D is called a cycle non-split vertex dominating set of G

Definition 3.6:

Let u and v be any two vertices of an IFG of G. Then u strongly dominates v (v weakly dominates u) if

(i) Strong arc between u and v

(ii) $d_N(u) \geq d_N$

A strong (weak) dominating set S of an IFG is said to be minimal strong (weak) dominating set if no proper subset of S is a strong (weak) dominating set of G. The minimum cardinality among all minimal **strong (weak) dominating set** is called strong (weak) intuitionistic fuzzy domination number of G, and is denoted γ_{sif} (G) (or) γ_{wif} (G).

Definition 3.6:

The set S is said to be **total dominating set** if for every vertex $v \in \gamma(G)$, V dominates to at least one vertex of S.

Theorem 3.1:

Let G be an ICFG without isolated edges and there exists no edge $e_i \in E$ such that $N_s(e_i) \subseteq D$. If D is a minimal edge dominating set, then S -D is an edge dominating set, where S is the set of all strong edge in G.

Proof:

Let *D* be a minimal edge dominating set of an ICFG *G*. Suppose *S -D* is not an edge dominating set. Then there exists at least one edge $e_i \in D$, which is not dominated by *S –D*.Since *G* has no isolated edges and there is no edge $e_i \in E$ such that $N_s(e_i) \subseteq D$, e_i is adjacent to at least one strong edge e_j in *S* . Since *S -D* is not an edge dominating set of *G*, $e_j \notin S - D$. Hence $e_j \in D$.

Therefore *D* - { e_i} is an edge dominating set which is contradiction to the fact that *D* is minimal edge dominating set. Thus, every edge in *E – S* is connected dominated by an edge in *S -D*.

Therefore *S -D* is an edge connected dominating set.

Hence the theorem proved.

Theorem 3.2:

If D is a point set dominating set in IFG G, with end edges, then at least one end edge occurs in D.

Proof:

Let D be an point connected dominating set in IFG G, with end edges. Suppose there is no end edge in D then some of the vertexes' in vertex dominating set D are independent and others are strong. This implies for each $\forall v_j \in V - D\ \exists v_i \in D$ such that v_i dominates to v_j but D will not be minimum. Then it is contradicts to the definition.

Therefore D contains at least one end edge in G. This completes the proof.

Theorem:3.3

Let D be a cycle non-split vertex dominating set, if ⟨V−D ⟩ edge cover includes all the edges of G.

Proof:

If know that D is a cycle non-split vertex IF dominating set $v \in D$.Assume that node cover of ⟨V−D ⟩ does not include all the edges of G. Then ⟨V−D⟩ may be connected (or) disconnected.

Case (i):

If ⟨V−D ⟩ is connected, then any two vertexes of vertex dominating set D contains no common edge and is disconnected. This implies D is a non-split vertex dominating set that contradicts the assumption that D is a cycle non-split dominating set.

Case (ii):

If ⟨V−D ⟩ is disconnected, then any two vertexes of vertex dominating set D are non-adjacent and there exists independent edges in D. Also at least one edge of end edge occurs in D. This implies vertex dominating set D is a split dominating set that is a contradiction to our assumption. Therefore from the above two cases, if conclude that edge cover of ⟨V−D ⟩ must contain all the edges of G.

This completes the proof.

Theorem 3.4:

If D is an vertex dominating set of ICFG G then at least one vertex dominating set D itself an vertex independent set.

Proof:

Let G be Intuitionistic connected fuzzy graph and D be an vertex Dominating set.

Case (i):

Let us assume that D contains an isolated vertex in G.

By definition of an Intuitionistic fuzzy vertex independent set, every v_i is an vertex independent set in G. obviously D is an vertex independent set.

Case (ii):

Suppose D is not an intuitionistic fuzzy vertex independent set. Then each vertex v_i in G will be strong. Thus we get G must be an Intuitionistic fuzzy graph with only strong arcs; this gives us contradiction to our assumption. Therefore at least one vertex dominating set must be an vertex independent set. This completes proof.

Theorem 4. 2:

Let D be a minimal TWIFD-set of an IFG. Then for each $v \in D$ iff the following holds.

(i) No vertex in D weakly dominates v.

(ii) There exists $u \in V - D$ such that v is the only vertex in D which weakly

Dominates u.

Proof:

Assume that D is a minimal TWIFD -set of G. Then for every

vertex $v \in D$, D-u is not a total weakly connected dominating set and hence there exist $u \in V - (D - \{v\})$ which is not weakly dominated by any vertex in $D - \{v\}$. If $u = v$, u is not weakly dominated by vertex in D,

If $v \neq u$, u is not weakly connected dominated by $D - \{u\}$, but u is weakly dominated by D Then the vertex u is weakly connected dominated by a vertex v in D.

Converse Part:

Assume that D is a Total strong dominating set and for each vertex $v \in D$, one of the two conditions holds.

(a) Suppose D is not minimal total weak dominating set, then there exists a

vertex $v \in D$, $D - \{v\}$ is a Total weak dominating set. Hence v is weak

dominated by at least one vertex in $D - \{v\}$, the condition one does not hold.

(b) If $D - \{v\}$ is a Total weak every vertex in $v - D$ is a weakly dominated by at least one vertex in $D - \{v\}$ the second condition does not holds.

Which is contradiction that at least one of these condition hold?

So, D is a minimal Total weakly dominating set.

Hence the theorem proved.

CONCLUSION:

Firstly the categorizeedge domination number of given graph in the Intuitionistic Fuzzy Environment. Further, The investigated the relationship between the edge &vertex domination number and other parameters such as edge dominating sets, end nodes, cut vertices and independent edge dominating sets. The concept of Total strong (weak) domination IFG is very rich both in theoretical developments and applications. In this paper, we introduced total strong (weak) domination IFG and some theorems proved.

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