INTUITIONISTIC FUZZY DECISION MAKING

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Abstract

Out of several generalizations of fuzzy set theory for various objectives, the notions of intuitionisticfuzzy sets is interesting and very useful in modeling real life problems. The ranking of fuzzynumbers was studied by many authors and it was extended to intuitionistic fuzzy sets becauseof its attraction and applicability. The ranking of intuitionistic fuzzy sets plays a vital role indecision-making, data analysis, artificial intelligence and socioeconomic system. In this papernew method for ranking intuitionistic fuzzy sets has been introduced and compared with aother methods by numerical examples.

Keywords: Intuitionistic fuzzy sets, Accuracy function, Arithmetic and Geometric aggregationoperators, Multicriteria fuzzy decision-making.

1. Introduction

The concept of fuzzy sets was introduced by Zadeh.Atanassov generalized this idea to intuitionistic fuzzy sets, and later there has been much progress in the study of intuitionistic fuzzy sets (IFSs).The notions of intuitionistic fuzzy numbers in different context were studied in .In the fuzzy set theory, the non membership grade of an element to lie in a fuzzy subset =1-membership grade of that the element to lie in the fuzzy subset. For example, if a person has 0.6 membership grade to lie in the fuzzy subset of good man, then according to the theory of fuzzy sets, he has 0.4 membership grade to lie in the fuzzy subset of not a being good man. But this is no longer true in real life problems.To overcome this difficulty, Atanassov (1986) defined a new notion of intuitionistic fuzzy sets.This concept generalizes the concept of fuzzy sets.

The ranking of fuzzy numbers and the ranking of intuitionistic fuzzy numbers plays a main role in real life problems involving imprecise information and incomplete information. Chen and Hwang score method was extended to intuitionistic fuzzy numbers in.The concept of score functions was introduced by Chen and Tan for vague values which are intuitionistic fuzzy values as pointed out by Deschrijver and Kerre

Subsequently, Hong and Choi indicated that the score function cannot discriminate some alternatives although they are apparently different and,hence , proposed an accuracy function and it was studied in X_U. In this paper we propose a new general score function to overcome this problem. This paper is organized as follows. Some preliminary definitions, a new score function in intuitionistic fuzzy set up, multi criteria fuzzy decision making methods are briefly introduce in section2. In section 3, illustrative example is given to show the validity of the new score function.

2. Preliminaries

Definition 1

Let X be a non empty set. An intuitionistic fuzzy set (IFS) A is of the form A= { $\langle X, \mu_A(X), \gamma_A((X)\rangle$:x$\in X$ } where the functions μ_A :X$\rightarrow$I and γ_A :X$\rightarrow I$

denote the degree of memb1ership and the degree of nonmembership of each element x $\in X$ to the set A, respectively, and 0+$\leq \mu_A$(x)+ γ_A(x)$\leq$1 for each x$\in$X.

For each element x we can compute the unknown degree (hesitancy degree) of $x \in X$ to lie in A which is defined as follows π_A(x)=1- μ_A(x)- γ_A(x).

The intuitionistic fuzzy value A(x) = (μ_A(x), γ_A(x)) has a physical interpretation, for example, if(μ_A(x), γ(x))=(0.4,0.3), then we can see that μ_A(x)=0.4 and γ_A(x)=0.3.It can be interpreted as the vote for resolution is 4 in favor,3 against ,and 3 absences.

Note

Intuitionistic fuzzy logic generalizes the concept of fuzzy logic.So any fuzzy subset is an intuitionistic fuzzy set and the converse is not true.We will denote the set of all the IFSs in X by IFS(X).

Definition2

Let A,B$\in$IFS(X).An incaution relation is defined by

$A \subset B \Leftrightarrow \mu_A(x)\leq \mu_B$(x) and γ_A(x)$\geq \nu_B$(x),$\forall$ x$\in$X.

Definition3

The equality of two IFS is defined by A=B$\Leftrightarrow A \subset B$ and $B \subset A$.

Definition4

Let A,B$\in$IFS(X).The addition A+B of A and B is defined by

$(A+B)(x)=(\mu_A(x)+\mu_B(x)-\mu_A(x)\mu_B(x), v_A(x)v_B(x))$.

Definition5

Let A,B$\in$IFS(X).The product AB of A and Bis defined by

$$(AB)(x)=(\mu_A(x)\mu_B(x), v_A(x)+v_B(x)-v_A(x)v_B(x)).$$

Definition6

Let $A_j(j=1,2\ldots.n)\in$IFS(X). The arithmetic average operator is defined by

$$F(A_1,A_2,\ldots A_n)=\sum_{j=1}^{n} A_j=\frac{1}{n}(1-\Pi(1-\mu_{A_j}(x)), \Pi(v_{A_j}(x))).$$

Definition7

Let $A_j(j=1,2\ldots n)\in$IFS(X).The geometric average operator is defined by

$$G(A_1,A_2\ldots A_n)=\Pi A_j=(\Pi\mu_{A_j}(x), 1-\Pi(1-v_{A_j})(x)).$$

Definition8

Let $A_j(j=1,2\ldots n)\in$IFS(X).The weighted arithmetic average operator is defined by

$$F_w(A_1,A_2\ldots A_n)=\sum_{j=1}^{n} w_j A_j=(1-\Pi(1-\mu_{A_j}(x))^{w_j}, \Pi(v_{A_j}(x))^{w_j}$$

Where w_j is the weight of $A_j(j=1,2\ldots n)$,$w_j\in[0,1]$ and $\sum_{j=1}^{n} w_j=1$.Especially ,assume $w_j=1/n(j=1,2\ldots n)$,then F_w is called an arithmetic average operator for IFSs.

Definition9

Let $A_j(j=1,2\ldots n)\in$IFS(X).The weighted geometric average operator is defined by

$G_W(A_1,A_2\ldots A_n)=\Pi A_j^{\;w_j}=(\Pi\mu_{A_j}(x)^{w_j}, 1-\Pi(1-v_{A_j}(x))^{w_j})$ where $w_j$is the weight of $A_j(j=1,2\ldots n)$,$w_j\in[0,1]$ and$\sum_{j=1}^{n} w_j=1$. Especially, assume $w_j=1/n(j=1,2\ldots n)$, then G_W is called an geometric average operator for IFSs.

The aggregated results $F_w$and G_W are still IFSs. Obviously, there are different emphases points between definitions 3.2.8 and 3.2.9.The weighted arithmetic average operator emphasizes the group's influence, so it is not very sensitive to $A_j(j=1,2\ldots n)\in$ IFS(X), where as the weighted geometric average operator emphasizes the individual influence, so it is more sensitive to $A_j(j=1,2\ldots n)\in$IFS(X).

Definition10

Let A=(μ_A, v_A) be an intuitionistic fuzzy value.The score function of A is given by S(A)= $\mu_A - v_A$.

Definition11

Let A_1= (μ_{A_1}, v_{A_1}) and A_2=(μ_{A_2}, v_{A_2})be intuitionistic fuzzy values.It is concluded that $A_1 \leq A_2$ if $S(A_1) \leq S(A_2)$ and vice versa.

Definition12

Let A= (μ_A, v_A) be an intuitionisticfuzzy value. The accuracyfunction of A is given by H (A) = $\mu_A + v_A$. Then it is concluded that $A_1 \leq A_2$ If $S(A_1)=(S_2)$ and $H(A_1) \leq H(A_2)$ and $A_1 \geq A_2$

If $S(A_1)=S(A_2)$ and $H(A_1) \succ H(A_2)$.If both score functions are same and accuracy function are same, then it is conclude that $A_1$and A_2 represent the same intuitionistic fuzzy information.

A New Score Function

Now we introduce a new score function

Definition13

Let A=(μ_A, V_A) be an intuitionistic fuzzy value. The new score function of A is given by N(A)= $\mu_A + \delta(1 - \mu_A - v_A)$ where $\delta \in [0,1]$ is a parameter depending on the decision maker's level of confidence.

when $\delta = 0$ N(A)= μ_A

when $\delta = 1$ N(A)=1- v_A

when $\delta = 1/2$ N(A)= $\frac{1 + \mu_A - v_A}{2}$

Ifthe decision maker is optimistic then he chooses $\delta = 1$and if the decision maker is pessimistic then he chooses $\delta = 0$.

3. Illustrative Example

In this section, an example for a multicriteria decision-making problem of alternatives is used as a demonstration of the application of the proposed fuzzy decision-making method in a realistic scenario.

There are four possible alternatives A_1,A_2,A_3 and A_4 of antibiotics. The doctor must take a decision according to the following three criteria:

(1) C_1 is the Fast Relief;
(2) C_2 is the Affordable;
(3) C_3 is the No Side effect.

The four possible alternatives are to be evaluated using the intuitionistic fuzzy information by the decision maker under the three criteria as listed in the following matrix.

	C_1	C_2	C_3
A_1	(0.4,0.4)	(0.4,0.2)	(0.1,0.6)
A_2	(0.6,0.3)	(0.6,0.3)	(0.4,0.2)
A_3	(0.3,0.4)	(0.5,0.4)	(0.4,0.3)
A_4	(0.7,0.2)	(0.6,0.3)	(0.3,0.2)

Using Geometric Average Operator

Here we obtain the geometric average value α_i for A_i(i=1,2,3,4) using definition 10 as follows

$$\alpha_1 = (\prod \mu_{A_I}(x), 1-\prod(1-\nu_{A_i})(x))$$

$$= ((\mu_{A_1}\mu_{A_2}\mu_{A_3}), 1-((1-v_{A_1})(1-v_{A_2})(1-\nu_{A_3})))$$

$$= ((0.4)(0.4)(0.1), 1-((1-0.4)(1-0.2)(1-0.6)))$$

$$=(0.016, 1-((0.6)(0.8)(0.4))$$

$$= (0.016, 1-0.192)$$

$$\alpha_1 = (0.016, 0.808).$$

$$\alpha_2 = (\prod \mu_{A_i}(x), 1-\prod(1-\nu_{A_i})(x))$$

$= ((\mu_{A_1}\mu_{A_2}\mu_{A_3}),1\text{-}((1\text{-}\nu_{A_1})(1-\nu_{A_2})(1-\nu_{A_3})))$

= ((0.6)(0.6)(0.4),1-((1-0.3)(1-0.3)(1-0.2)))

=(0.144,1-((0.7)(0.7)(0.8))

=(0.144,1-0.392)

α_2=(0.144,0.608).

Similarly,

α_3=(0.06,0.748), α_4=(0.126,0.552).

By applying definition 13 we get N(α_i)(i=1,2,3,4)as

N(α_1)= $\mu_A+\delta(1-\mu_A-\nu_A)$

=0.016+ δ (1-0.016-0.808)

N(α_1)=0.016+ δ (0.176).

N(α_2)= $\mu_A+\delta(1-\mu_A-\nu_A)$

=0.144+ δ (1-0.144-0.608)

N(α_2)=0.144+ δ (0.248).

Similarly,

N(α_3)=0.06+ δ (0.192), N(α_4)=0.126+ δ (0.322).

Here the alternatives are ranked as A_2>A_4>A_3>A_1 whenever δ <0.2432 and as A_4>A_2>A_3>A_1 whenever δ <0.2432.

Using Weighted Geometric Average Operator

Assuming that the weights of C_1,C_2 and $C_3$are 0.35,0.25 and 0.40,we obtain the weighted geometric average value α_i for A_i (i=1,2,3,4) using definition 9 as follows

$\alpha_1 = (\prod \mu_{A_{j(}}(x)^{w_j},1-\prod(1-\nu_{A_j}(x))^{w_j})$

$=((\mu_{A_1}{}^{w_1}\mu_{A_2}{}^{w_2}\mu_{A_3}{}^{w_3}),1-(1-\nu_{A_1})^{w_1}(1-\nu_{A_2})^{w_2}(1-\nu_{A_3})^{w_3})$

$=((0.4^{0.35})(0.4^{0.25})(0.1^{0.40}),1\text{-}(1\text{-}0.4)^{0.35}(1\text{-}0.2)^{0.25}(1\text{-}0.6)^{0.40})$

α_1=(0.2297,0.4517)

Similarly,

α_2=(0.5102,0.2616),

α_3=(0.3824,0.3618),

α_4=(0.4799,0.2263).

By applying definition 13,we get N(α_i)(i=1,2,3,4) as

$N(\alpha_1) = \mu_A + \delta(1 - \mu_A - v_A)$

=0.2297+δ(1-0.2297-0.4517)

N(α_1)=0.2297+δ(0.3186).

Similarly,

N(α_2)=0.5102+δ(0.2282),

N(α_3)=0.3824+δ(0.2558),

N(α_4)=0.4799+δ(0.2938).

Here the alternatives are ranked as $A_2>A_4>A_3>A_1$ whenever δ<0.4618 and as $A_4>A_2>A_3>A_1$ whenever $\delta \geq 0.4618$.

Using Weighted Arithmetic Average Operator

Here we can obtain arithmetic average value α_i for A_i(i=1,2,3,4) using definition6as follows

$$\alpha_1 = \frac{1}{n}(1 - \prod(1 - \mu_{A_j}(x)), \prod(v_{A_j}(x)))$$

=1/4(1-(1-0.4)(1-0.4)(1-0.1),((0.4)(0.2)(0.6)))

=(0.25)(0.324,0.048)

α_1=(0.169,0.012).

Similarly,

α_2=(0.226,0.0045)

α_4=(0.229,0.003).

By applying definition 13,we get N(α_i)(i=1,2,3,4) as

N(α_1)= $\mu_A + \delta(1-\mu_A - v_A)$

=0.169+δ(1-0.169-0.048)

N(α_1) =0.169+δ(0.819)

Similarly,

N(α_2)=0.226+δ(0.7695)

N(α_3)=0.1975+δ(0.7905)

N(α_4)=0.229+δ(0.768).

Here the alternatives are ranked as $A_4 > A_2 > A_3 > A_1$ for some $\delta < 1$

Using Weighted Arithmetic Operator

Assuming the same weight for C_1,C_2 and C_3,we obtain the weighted arithmetic average value α_i for A_i (i=1,2,,3,4)using definition8as follows

$$\alpha_1 = (1-\prod(1-\mu_{A_j}(x))^{w_j}, \prod(v_{A_j}(x))^{w_j}$$

$$= (1-(1-0.4)^{0.35}(1-0.4)^{0.25}(1-0.1)^{0.40},(0.4^{0.35})(0.4^{0.25})(0.1^{0.40}))$$

=(1-0.70576,0.3956)

α_1=(0.2943,0.3956)

Similarly,

α_2=(0.5296,0.2551)

α_3=(0.3949,0.3565)

α_4=(0.5476,0.2213).

By applying definition 13,we get N(α_i)(i=1,2,3,4) as

N(α_1)= $\mu_A + \delta(1-\mu_A - v_A)$

$= 0.2943+\delta(1\text{-}0.2943\text{-}0.3956)$

$N(\alpha_1)=0.2943+\delta(0.3101)$

Similarly,

$N(\alpha_2)=0.5296+\delta(0.2153)$

$N(\alpha_3)=0.3949+\delta(0.2486)$

$N(\alpha_4)=0.5476+\delta(0.2311)$.

Here the alternatives are ranked as $A_4>A_2>A_3>A_1$ for all values of δ.

Conclusion

The alternatives are ranked as $A_4>A_2>A_3>A_1$ when $\delta<0.2432$

(Using geometric operator) and $\delta<0.4618$ (using weighted geometric operator).

The alternatives are ranked as $A_4>A_2>A_3>A_1$ when $\delta>0.4618$ for all the four average operators (geometric average operator, weighted geometric average operator, arithmetic average operator, weighted arithmetic average operator).

Hence it can be concluded that $A_4> A_2> A_3> A_1$ is the optimal ranking.

In this paper, the multi criteria decision making under IFS set up is studied. This can also be extended to interval valued intuitionistic fuzzy set up. Since the problem of ranking of intuitionistic fuzzy is very much important in real life problems such as decision making, clustering and artificial intelligence, this study is very much useful and has wide applications in all areas.

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